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Unification of the streamline, heatline and massline methods for the visualization of two-dimensional transport phenomena

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Abstract

Functions and lines used for visualization purposes can be unified from physical and numerical viewpoints. The physical unification represents an important step, in order to make common the procedure used to obtain the differential equation from which are obtained the functions whose contour plots (lines) are used for visualization purposes. From the numerical viewpoint, the unification results are also very interesting, making possible the evaluation of the functions' fields by using the same numerical procedures and code routines as for the primitive conserved variables. For domains involving media with very different properties, the harmonic mean practice has been shown to be the most attractive procedure to evaluate the interfacial diffusion coefficient, both for the primitive conserved variables and for the functions introduced for visualization purposes. © 1998 Published by Elsevier Science Ltd. All rights reserved.

Nomenclature

A area
A, B mesh points
B wall thickness
C concentration
c_p constant pressure specific heat
D mass diffusion coefficient
g gravitational acceleration
H heatfunction
i control volume interface
$\mathbf{i}$, $\mathbf{j}$ Cartesian unit vectors
$\mathbf{J}$ transport flux vector
k thermal conductivity
L length
Le Lewis number
M massfunction
N buoyancy ratio
$\mathbf{n}$ outward unit normal
p pressure
Pe Péclet umber
Pr Prandtl number
Ra Rayleigh number
Rc thermal conductivity ratio
Rd mass diffusion coefficient ratio
s segment
S source term
T temperature
u, v velocity components
V velocity
x, y Cartesian co-ordinates.

Greek symbols

α thermal diffusivity
β volumetric expansion coefficient
Γ generic diffusion coefficient
Δ distance or difference value
ε constant small number
μ dynamic viscosity
ν kinematic viscosity
ρ density
ϕ generic intensive (specific) property
Φ generic function related to the ϕ property
ψ stream function.

Subscripts

C cold wall or mass effects
i interface value or referring to i species
f fluid medium
H higher value

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ref reference value
T referring to thermal effects
w wall medium
0 reference value (lower value)
* dimensionless.

1. Introduction

Streamfunction and streamlines are very efficient and largely used tools to visualize two-dimensional flow fields [1]. In the study of conduction heat transfer, the use of the heat flux lines is well established [2]. In the field of convection heat transfer, the heatfunction and heatline concepts were introduced by Kimura and Bejan [3], and also by Bejan [4] in the last decade only. Examples of heatline applications can be found in some recent literature [3–15]. A natural extension was made for the field of convective mass transfer, by introducing the massfunction and massline concepts, which can be found also in some recent literature [15–17]. From a physical viewpoint, the used functions are usually treated individually, each requiring a particular procedure. The same is also valid from a numerical viewpoint, the functions used for visualization being obtained using special procedures and routines, other than those used for the calculation of the primitive fields.

The main objective of this work is to present a general physical and numerical treatment for functions and lines used for visualization purposes in two-dimensional situations. From the numerical viewpoint, it is shown that the same procedures and code routines involved in the calculation of the primitive fields can also be used in the evaluation of the functions fields, even on situations involving conjugated transfer phenomena. It is assumed that such functions are used for visualization purposes only, their fields being evaluated once the fields of the primitive variables velocity, pressure, temperature and concentration are known.

2. Physical modeling

The usual two-dimensional heat, mass and related transfer phenomena are described by partial differential equations, which can be written in the general conservative form [18]

$$\frac{\partial}{\partial x}(\rho u\phi)+\frac{\partial}{\partial y}(\rho v\phi)=\frac{\partial}{\partial x}\left(\Gamma_\phi\frac{\partial\phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma_\phi\frac{\partial\phi}{\partial y}\right)+S_\phi, \tag{1}$$

whose solutions can be obtained by using many powerful numerical methods available. ϕ is the specific transported variable, and some particular meanings of ϕ are presented in Table 1, special emphasis being given to the particular diffusion coefficients and source terms.

Table 1
Diffusion coefficients and source terms for each individual ϕ in equation (1)

Physical principle	ϕ	Γ_ϕ	S_ϕ
Overall mass conservation	1	0	0
x momentum equation	u	μ	$-\partial p/\partial x+\ldots$
y momentum equation	v	μ	$-\partial p/\partial y+\ldots$
Energy conservation	T	k/c_p	0
i species mass conservation	C_i	ρD_i	0

If the fluid flow subsides (stagnant fluid or solid medium with $u = v = 0$), the corresponding diffusion situation is described by the right hand side of equation (1), k/c_p and ρD_i being, in that case, the heat and i species mass diffusion coefficients over the involved medium. The global mass conservation ($\phi = 1$), with null source term and diffusion coefficient, is valid for any medium if nuclear reactions are not present, as is the usual case. By their turn, the x and y momentum equations are of application if the fluid flow subsists, and their source terms can include any body force (by unit mass) in addition to the negative of the pressure gradient components, such as buoyancy terms. The presented energy equation is valid if there are no source or sink terms, and the i species mass conservation equation is valid if there is no i species production or destruction (no chemical reactions involving i species). The general unification procedure to be developed applies only to differential equations, in the form of equation (1), with *no source terms*. If the differential equation for a given variable presents a non-zero source term, the developed visualization tools are of no value for such a variable.

Equation (1), with no source term, can be written as

$$\frac{\partial}{\partial x}\left[\rho u(\phi-\phi_0)-\Gamma_\phi\frac{\partial\phi}{\partial x}\right]+\frac{\partial}{\partial y}\left[\rho v(\phi-\phi_0)-\Gamma_\phi\frac{\partial\phi}{\partial y}\right]=0, \tag{2}$$

noting that the terms involving ϕ_0 vanish by invoking the differential mass conservation equation, that is, $[\partial/\partial x(\rho u)+\partial/\partial y(\rho v)]\phi_0 = 0$. The meaning of ϕ_0, as well as the reason for its introduction, will be explained below.

In equation (2) the $\mathbf{J}_\phi$ flux components are identified as

$$J_{\phi,x}=\rho u(\phi-\phi_0)-\Gamma_\phi\frac{\partial\phi}{\partial x}$$

$$J_{\phi,y}=\rho v(\phi-\phi_0)-\Gamma_\phi\frac{\partial\phi}{\partial y}. \tag{3}$$

Defining now the function $\Phi(x, y)$, through its first order derivatives, as

$$\frac{\partial \Phi}{\partial y} = \rho u(\phi - \phi_0) - \Gamma_\phi \frac{\partial \phi}{\partial x}$$
$$-\frac{\partial \Phi}{\partial x} = \rho v(\phi - \phi_0) - \Gamma_\phi \frac{\partial \phi}{\partial y}, \tag{4}$$

equation (1) can be obtained by equating the second order crossed derivatives of Φ, being implicitly assumed that $\Phi(x, y)$ is a continuous function to its second order derivatives.

The total differential of $\Phi(x, y)$ is obtained as

$$\mathrm{d}\Phi = \frac{\partial \Phi}{\partial x}\mathrm{d}x + \frac{\partial \Phi}{\partial y}\mathrm{d}y = -J_{\phi,y}\,\mathrm{d}x + J_{\phi,x}\,\mathrm{d}y, \tag{5}$$

that is, $\mathrm{d}\Phi = \mathbf{J}_\phi \cdot \mathbf{n}\mathrm{d}A$, with $\mathrm{d}A = \mathrm{d}s \times 1$, the $\mathrm{d}s \times 1$ product being presented to emphasize that we are considering the two-dimensional situation of Fig. 1 with a unit depth. If $\mathrm{d}\Phi = 0$, it means that there is not any ϕ flow crossing segment $\mathrm{d}s$ in Fig. 1, that is, a Φ constant line is a non-crossed line by the ϕ flow, being thus a line that is tangent to the flow vector. There are such constant Φ lines that are of major importance for visualization purposes.

It should be noted that a difference $\Delta\Phi$ between the Φ values at two points represents the ϕ flow that, by unit depth, crosses the segment linking these points, being thus specially instructive the streets comprised between two constant Φ lines, in which well bordered ϕ flows are transferred.

The ϕ_0 value is introduced by the fact that, usually, any variable other than the pressure or the velocity components is made dimensionless as $\phi_* = (\phi - \phi_0)/(\phi_H - \phi_0)$, ϕ_H and ϕ_0 being, respectively, the higher and lower values of ϕ in the domain [15]. For the continuity equation, which involves only the velocity components that are made dimensionless as $(u_*, v_*) = (u, v)/V_{\mathrm{ref}}$, the ϕ_0 value is not taken as $\phi_0 = 1$ but $\phi_0 = 0$. Thus, the first order derivatives of Φ can be made dimensionless, for a constant property situation, as

$$\frac{\partial \Phi_*}{\partial y_*} = Pe_\phi(u_*\phi_*) - \frac{\partial \phi_*}{\partial x_*}$$
$$-\frac{\partial \Phi_*}{\partial x_*} = Pe_\phi(v_*\phi_*) - \frac{\partial \phi_*}{\partial y_*}, \tag{6}$$

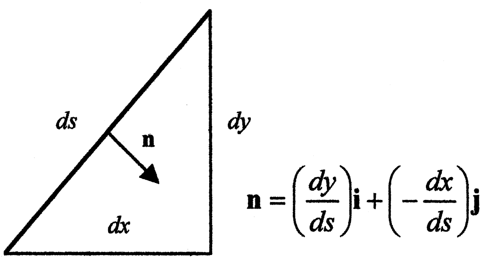

Fig. 1. Elementary segment $\mathrm{d}s = \sqrt{\mathrm{d}x^2 + \mathrm{d}y^2}$ crossed by the flux $\mathbf{J}_\phi$.

where $Pe_\phi = \rho V_{\mathrm{ref}} L/\Gamma_\phi$, L being the characteristic length, and $\Phi_* = \Phi/\Gamma_\phi(\phi_H - \phi_0)$. For $\phi = 1$, the continuity equation, $\Phi = \psi$, the streamfunction, the first order derivatives can be made dimensionless as $\partial\psi_*/\partial y_* = u_*$ and $-\partial\psi_*/\partial x_* = v_*$, with ψ_* defined as $\psi_* = \psi/\rho V_{\mathrm{ref}} L$. In all that follows, it will be considered that $\Gamma_\phi = \varepsilon$ for the continuity equation, ε being a constant small number.

Assuming now that ϕ is a continuous function to its second order derivatives, the equality of its second order cross derivatives can be established through the expressions obtained from the right hand sides present in equation (4), leading to the equation

$$0 = \frac{\partial}{\partial x}\left(\frac{1}{\Gamma_\phi}\frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{1}{\Gamma_\phi}\frac{\partial \Phi}{\partial y}\right) + \left\{\frac{\partial}{\partial x}\left[\frac{\rho v}{\Gamma_\phi}(\phi - \phi_0)\right] - \frac{\partial}{\partial y}\left[\frac{\rho u}{\Gamma_\phi}(\phi - \phi_0)\right]\right\}. \tag{7}$$

This is the second order partial differential equation from which it will be evaluated the Φ field, for any particular corresponding meaning of ϕ. It is an equation corresponding to a conduction-type problem, with source term if the fluid flow subsists and without source term if the fluid flow subsides, with the diffusion coefficient for Φ verifying

$$\Gamma_\Phi = 1/\Gamma_\phi, \tag{8}$$

which is maintained within parenthesis in equation (7) because it is, in the general case, a variable and not a constant. To the authors' knowledge, this is the first formulation considering a variable diffusion coefficient for Φ. For $\phi = 1$, $\phi_0 = 0$ and $\Gamma_\phi = \varepsilon$, a small constant number, one obtains the well-known partial differential equation for the streamfunction [1]. The particular meaning of Φ for some usual situations is summarized in Table 2.

Equation (7) is a conduction-type equation and, for each particular Φ, its solution can be obtained following the same procedures as for ϕ, once the boundary conditions are established. From a physical viewpoint, we have thus a unified treatment for the functions used for visualization purposes.

The Φ function is defined through its first order derivatives, equation (4), being thus only important differences on the Φ values but not the Φ level. This relative behavior is similar to that of the pressure when evaluating incompressible fluid flows, with a total freedom to choose any suitable reference point. The Φ field is evaluated once its corresponding ϕ field is known, the Φ values over the boundaries being obtained by integrating the adequate Φ derivative present in equation (4) through the boundaries. Extending this procedure to all the domain boundaries, starting from any suitable reference point, we have boundary conditions of first kind for Φ over all the domain boundary. Over the vertical (along y) boundaries one obtains

Table 2
Coupling of ϕ and Φ for some usual situations

Physical principle	ϕ	Φ	Φ contour plots
Overall mass conservation	1	ψ—Streamfunction	Streamlines
Energy conservation	T	H—Heatfunction	Heatlines
i species mass conservation	C_i	M_i—i species massfunction	i species masslines

$$\Phi(x_{\text{ref}}, y) = \Phi(x_{\text{ref}}, y_{\text{ref}}) + \int_{y_{\text{ref}}}^{y} \left[\rho u(\phi - \phi_0) - \Gamma_\phi \frac{\partial \phi}{\partial x} \right] \mathrm{d}y, \tag{9a}$$

and over the horizontal (along x) boundaries the corresponding integration is

$$\Phi(x, y_{\text{ref}}) = \Phi(x_{\text{ref}}, y_{\text{ref}}) - \int_{x_{\text{ref}}}^{x} \left[\rho v(\phi - \phi_0) - \Gamma_\phi \frac{\partial \phi}{\partial y} \right] \mathrm{d}x. \tag{9b}$$

3. Numerical modeling

From a numerical viewpoint, equation (7) is a conduction-type equation for the Φ variable, with or without source term depending if the fluid flow subsists or not. Its solution, for each particular meaning of Φ, is easier than that of its corresponding ϕ. The most important aspect from the unification viewpoint is that the same numerical procedures and routines used for the primitive variables ϕ can be used in order to evaluate the corresponding Φ fields.

The diffusion coefficient for Φ can be treated through any suitable practice similar to that used for the ϕ diffusion coefficient. However, if a control volume finite difference method is used, the harmonic mean practice [18] shows to be the most attractive one. At this point, it should be noted that if we are evaluating the Φ field by using a control volume method, we are assuming that Φ is a *conserved variable*, which is not necessarily the case. This is not of major importance, due to the fact that the obtained information is used specially for visualization purposes, with an essentially qualitative value. The referred harmonic mean practice is the exact one for one-dimensional situations, and it is the most suggestive one for the situations with sharp variations in the involved diffusion coefficients, as is the case of conjugated transport phenomena with a domain composed by contiguous and very different materials.

Considering the one-dimensional situation sketched in Fig. 2, representing a conduction situation, the interface diffusion coefficients, obtained by using the harmonic mean practice, are

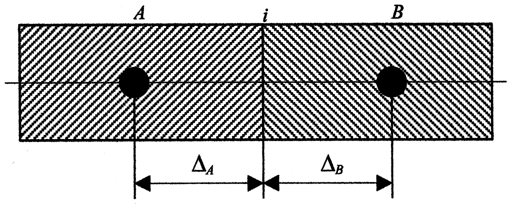

Fig. 2. Numerical cells to apply the harmonic mean practice.

$$\Gamma_{\phi,i} = \frac{\Gamma_{\phi,A}\Gamma_{\phi,B}(\Delta_A + \Delta_B)}{\Gamma_{\phi,A}\Delta_B + \Gamma_{\phi,B}\Delta_A} \quad \Gamma_{\Phi,i} = \frac{(\Delta_A + \Delta_B)}{\Gamma_{\phi,A}\Delta_A + \Gamma_{\phi,B}\Delta_B} \tag{10}$$

and the corresponding ϕ and Φ interface values are

$$\phi_i = \frac{(\Gamma_{\phi,A}\Delta_B)\phi_A + (\Gamma_{\phi,B}\Delta_A)\phi_B}{\Gamma_{\phi,A}\Delta_B + \Gamma_{\phi,B}\Delta_A}$$

$$\Phi_i = \frac{(\Gamma_{\phi,B}\Delta_B)\Phi_A + (\Gamma_{\phi,A}\Delta_A)\Phi_B}{\Gamma_{\phi,B}\Delta_B + \Gamma_{\phi,A}\Delta_A}. \tag{11}$$

The limit situations for $\Gamma_{\phi,i}$, $\Gamma_{\Phi,i}$, ϕ_i and Φ_i corresponding to $\Gamma_\phi \to 0^+$ or $\Gamma_\phi \to +\infty$ are shown in Table 3.

From Table 3, if $\Gamma_{\phi,A} \to 0^+$ (material A with null diffusivity) one obtains $\Gamma_{\phi,i} = 0$, that is, there is no diffusion at the interface, and any constant ϕ line is normal to the interface. By this turn, the ϕ value at the interface is this corresponding to node B. Considering now the situation of $\Gamma_{\phi,A} \to +\infty$, being thus A a $\phi = \phi_A$ constant layer, one finds that the diffusion coefficient at the interface is dominated by $\Gamma_{\phi,\mathrm{B}}$, and any constant ϕ line is parallel to the interface. The ϕ value at the interface is ϕ_A in this case. Similar results can be obtained analysing the $\Gamma_{\phi,B}$ limit situations. It should be stressed that the situation of $\Gamma_{\phi,A} \to 0^+$ and $\Gamma_{\phi,B} \to 0^+$ is of no sense, because the layers A and B are not in communication through any transfer phenomena in this case, as well as the situation of $\Gamma_{\phi,A} \to +\infty$ and $\Gamma_{\phi,B} \to +\infty$, which corresponds to a constant ϕ value over both layers A and B.

The foregoing limit situations are analysed now from the viewpoint of the Φ function. If $\Gamma_{\phi,A} \to 0^+$, that is, $\Gamma_{\Phi,A} \to +\infty$, one finds that the diffusion coefficient $\Gamma_{\Phi,i}$ at the interface is dominated by $1/\Gamma_{\phi,B}$, layer A is a $\Phi = \Phi_A$ constant region, and any constant Φ line is parallel to the

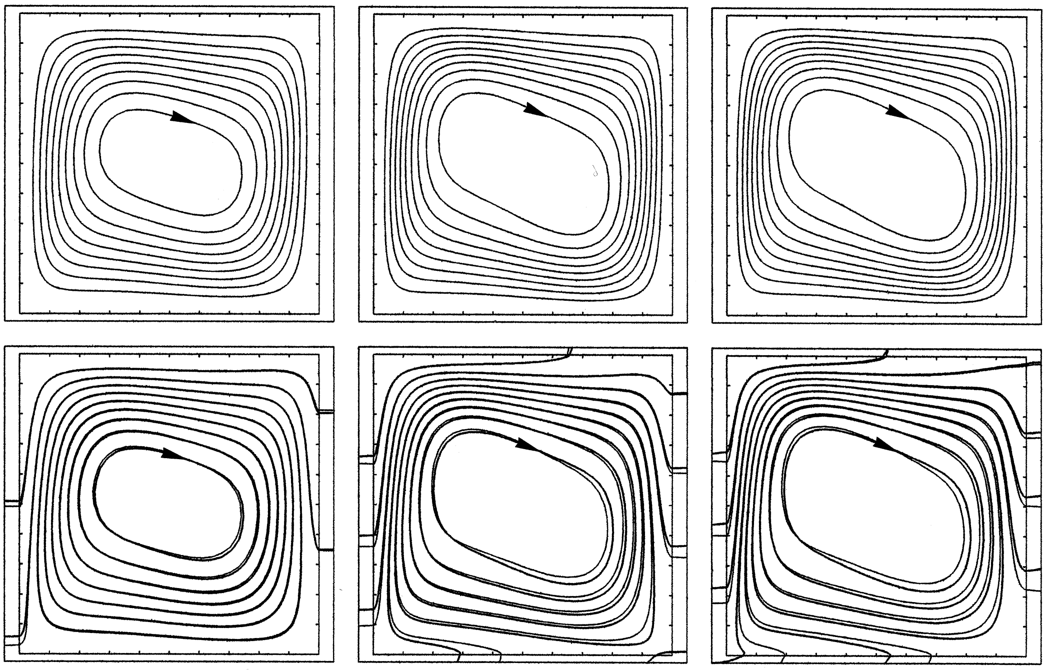

Fig. 3. Streamlines (top), and heatlines (——) and masslines (——) (bottom) for combined buoyancy effects and $N = 0.5$: (a) $Rc = Rd = 0.1$ ($\Delta\psi_* = 0.788$; $\Delta H_* = 0.455$; $\Delta M_* = 0.455$); (b) $Rc = Rd = 0.5$ ($\Delta\psi_* = 1.137$; $\Delta H_* = 0.765$; $\Delta M_* = 0.761$); and (c) $Rc = Rd = 1.0$ ($\Delta\psi_* = 1.259$; $\Delta H_* = 0.896$; $\Delta M_* = 0.888$).

interface. The Φ value at the interface is Φ_A in this case. For the situation of $\Gamma_{\phi,A} \to +\infty$, that is, $\Gamma_{\Phi,A} \to 0^+$, the diffusion coefficient $\Gamma_{\Phi,i}$ at the interface is null, and any constant Φ line is normal to the interface. The Φ value at the interface is Φ_B in this case. Once again, similar results can be obtained analysing $\Gamma_{\phi,B}$ limit situations.

As we are analysing the near boundary region considering a simple conduction model, we obtain a result similar to that obtained when using the heat flux lines on single conduction problems, involving isotropic media, the heatlines being normal to the isothermals.

It should be noted that the harmonic mean practice is used, but the inverse of the harmonic mean is not the harmonic mean of the inverse, as it can be easily observed from Table 3. Thus, the corresponding diffusion coefficients for ϕ and Φ must be obtained by using the harmonic mean practice, from the respective nodal diffusion coefficients, ($\Gamma_{\phi,A}$, $\Gamma_{\phi,B}$) and ($\Gamma_{\Phi,A}$, $\Gamma_{\Phi,B}$).

The overall mass transfer equation, $\phi = 1$, is only a normal case to apply the here proposed unified procedure. For a domain delimited by some solid impermeable walls, we have $u = v = 0$ and $\Gamma_\phi \to 0^+$, that is $\Gamma_\Phi \to +\infty$ over such walls. The simplest solution of equation (7) for $\phi = 1$ over the solid walls (with null source term) is $\Phi = \Phi_{\rm wall} =$ constant. The wall as well as the fluid layer close to the wall is a $\Phi = \Phi_{\rm wall}$ constant region, being any constant Φ line parallel to the fluid-solid interface. In this case, $\phi = 1$, we have $\Phi = \psi$, the streamfunction, which is effectively parallel to any (perfectly) impermeable boundary, being usually considered $\psi_{\rm wall} = 0$. Over the fluid flowing, the source term of equation (7) is not null, the diffusion coefficient can be eliminated from the differential equation as it is constant, $1/\Gamma_\phi = 1/\varepsilon$, and the ψ solution corresponding to the flowing fluid is conjugated with the $\psi = \psi_{\rm wall} = 0$ distribution over the confining solid walls.

4. Illustrations

In order to show the capabilities of the proposed unifying procedure, some results concerning the double-diffusive natural convection in a square enclosure with heat and mass diffusive walls are presented [19]. It is a square enclosure with $L/B = 20$, B being the wall thickness, filled with moist air ($Pr = \nu/\alpha = 0.7$, $Le = \alpha/D = 0.8$) and $Ra_{\rm T} = g\beta_{\rm T}L^3(T_{\rm H}-T_{\rm C})/\nu\alpha = 10^5$.

In the vertical direction, it is assumed a vertical stack of equal square enclosures separated by walls of thickness B, and a periodic spatial variation in this direction. The remaining important parameters are the buoyancy ratio $N = \beta_{\rm C}(C_{\rm H}-C_{\rm C})/\beta_{\rm T}(T_{\rm H}-T_{\rm C})$, the heat and mass

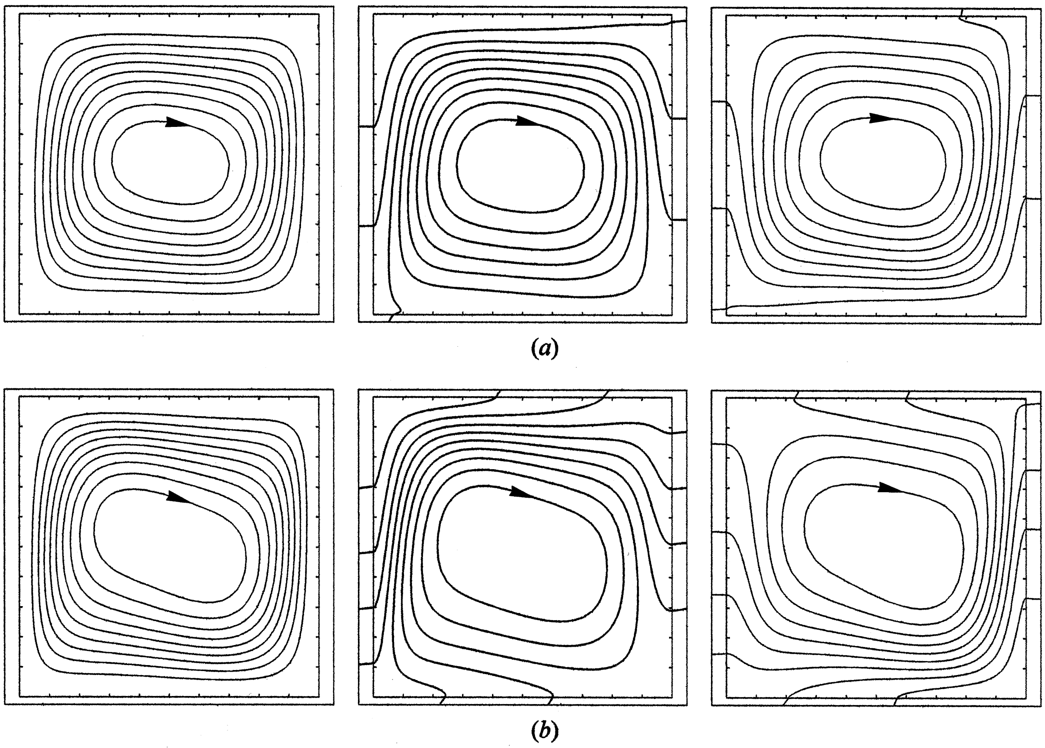

Fig. 4. Streamlines (left), heatlines (center) and masslines (right) for opposed buoyancy effects and $N = 0.5$: (a) $Rc = Rd = 0.1$ ($\Delta\psi_* = 0.485$; $\Delta H_* = 0.300$; $\Delta M_* = 0.300$); and (b) $Rc = Rd = 1.0$ ($\Delta\psi_* = 0.783$; $\Delta H_* = 0.599$; $\Delta M_* = 0.594$).

Table 3
Limiting situations resulting from the use of the harmonic mean practice

Limit situation	$\Gamma_{\phi,i}$	$\Gamma_{\Phi,i}$	ϕ_i	Φ_i
$\Gamma_{\phi,A} \to 0^+$	0	$\frac{\Delta_A+\Delta_B}{\Delta_B}\frac{1}{\Gamma_{\phi,B}}$	ϕ_B	Φ_A
$\Gamma_{\phi,A} \to +\infty$	$\Gamma_{\phi,B}\frac{\Delta_A+\Delta_B}{\Delta_B}$	0	ϕ_A	Φ_B
$\Gamma_{\phi,B} \to 0^+$	0	$\frac{\Delta_A+\Delta_B}{\Delta_A}\frac{1}{\Gamma_{\phi,A}}$	ϕ_A	Φ_B
$\Gamma_{\phi,B} \to +\infty$	$\Gamma_{\phi,A}\frac{\Delta_A+\Delta_B}{\Delta_A}$	0	ϕ_B	Φ_A

diffusion coefficient ratios $Rc = k_w/k_f$ and $Rd = (\rho D)_w/(\rho_0 D)_f$, and the combined or opposed heat and mass buoyancy effects. The thermal and mass buoyancy effects are modeled through a Boussinesq type approach, the density being a constant $\rho = \rho_0$ except on the vertical velocity source term, where it is a function of both the temperature and moisture concentration fields. The involved functions for visualization purposes are the streamfunction, the heatfunction, and the massfunction, the later relative to the humidity present in the moist air filling the cavity.

Analysis of Figs 3 (combined buoyancy effects) and 4 (opposed buoyancy effects) shows all the potential of the unifying procedure proposed in this work, together with the use of the harmonic mean practice for the evaluation of the interface diffusion coefficients.

5. Conclusions

As the main conclusion of this work, the heat and mass functions conjugated problems, as well as the streamfunction problem, can be derived using a common physical procedure, and such problems are formally similar. If the physical unification is interesting, another not less important conclusion is that these problems can be numerically solved by using the same procedures and routines used when solving for the primitive conserved variables, including the harmonic mean practice for the diffusion coefficients. By their own turn, the lines obtained as contour plots of such functions show that they are an effective way to visualize the involved transport phenomena, instead of the intensively used contour plots of the primitive variables such as pressure, temperature and concentration. Contour plots of the primitive variables are important to visualize the levels of the variables through the domain, but not to visualize the involved transport phenomena. Contrarily, contour plots of the introduced Φ functions are important to visualize the involved transport phenomena, but not the level of the primitive variables through the domain.

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